Theoretical Distribution

Distribution of Total probability on the basis of a random variable which can be discrete or continous.

So we can say that Theoretical Distribution is a mathematical function that describes all the possible values of a random variable & their probabilities

If Random variable is descrete then Probability distribution is known as probability mass function

eg

x_i	p_i
1	0.2
2	0.4
3	0.3
4	0.1
	1

x is descrete Random variable

like: No of accidents, No of goals in a football match & No of Defective bulbs

In this chapter we will study about two descrete prob. Distribution

1) Binomial Distribution
2) Poisson Distribution

If Random variable is continous Then prob. Distribution is known as prob. Density function

eg x_i: Height, Age, wages, salary.

CI	P_i
0-10	0.2
10-20	0.5
20-30	0.1
30-40	0.2
	1

In this chapter we will study only one continous prob. Distribution which is "Normal Distribution"

Binomial Distribution

→ Also known as Bernoulli Trials

→ According To Bernoulli

→ If an experiment is performed 'n' times Then n = No. of Trials

→ No of Trials is a finite positive integer

→ outcomes of each trials are categorized as success (p) & failure (q)

→ In each trial prob. of success & failure remain same (Trials are independent)

→ $p+q=1$ & $q=1-p$

→ If x is a random variable then $x=0,1,2,3,\dots,n$

$$x \sim B(n, p)$$

$$P(x=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

for $r=0,1,2,3,\dots n$

$$P(x=0)+P(x=1)+\dots+P(x=n)=1$$

→ Binomial Distribution is biparametric, There are two Parameters n & p

→ mean (μ) = np

→ variance (σ^2) = npq

→ S.D. (σ) = $\sqrt{npq}$

→ maximum variance = $\frac{n}{4}$

when $p = q = \frac{1}{2}$

→ Additive property

if $X \sim B(n_1, p)$

$Y \sim B(n_2, p)$

Then $x + y \sim B(n_1 + n_2, p)$

→ mode of Binomial Distribution Depends on the value of $(n+1)p$.

If $(n+1)p$ is non integer
Then mode $= [(n+1)p]$ greatest integer

If $(n+1)p$ is integer
Then there are two modes
First mode $= (n+1)p$
second mode $= (n+1)p - 1$

→ This used when 'n' is small & p is not small

Poisson Distribution

for descrete Random variable

→ used when prob. of success in a small time interval is very small.

→ n is big & p is small

→ mean = m = np

→ Poisson Distribution is uniparametric

m is the only parameter

→ for Random variable x

$x \sim P(m)$

$$P(X=r) = \frac{e^{-m} m^{r}}{r!}$$

for $r = 0, 1, 2, 3, \ldots \infty$

→ $P(X=0) + P(X=1) + P(X=2) + \dots = 1$

→ $e = 2.7183$

→ $e^{x} = AL(x \times 0.4343)$

→ mean $= m = np$

→ variance $= m = np$

→ S.D. $= \sqrt{m}$

→

$$\text{mode} = \begin{cases} [m] \text{ integral part} & \text{If } m \text{ is non integer} \\ m \ \& \ m-1 & \text{If } m \text{ is integer} \end{cases}$$

→ Additive property

If $X \sim P(m_1)$ & $Y \sim P(m_2)$

Then $X + Y \sim P(m_1 + m_2)$

Normal Distribution

⇓

Also known as Gaussian Distribution

⇓

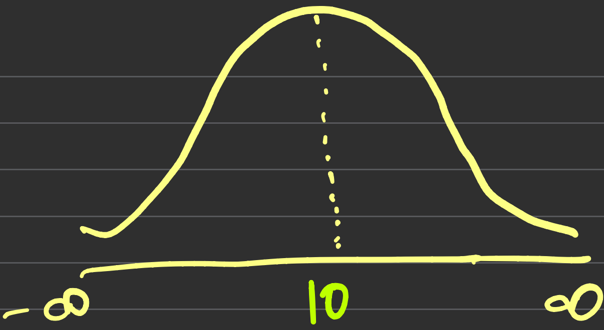
most important & universally accepted continous prob. Distribution function

It is a biparametric Distribution

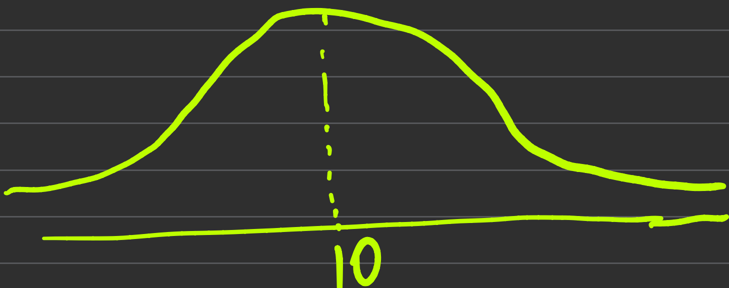
mean(μ) & variance (σ^2) are two parameters which decides the shape of normal Distribution curve

μ ⇒ It will tell central value

σ^2 ⇒ It decides the spread



$\mu = 10$
$\sigma^2 = 2$

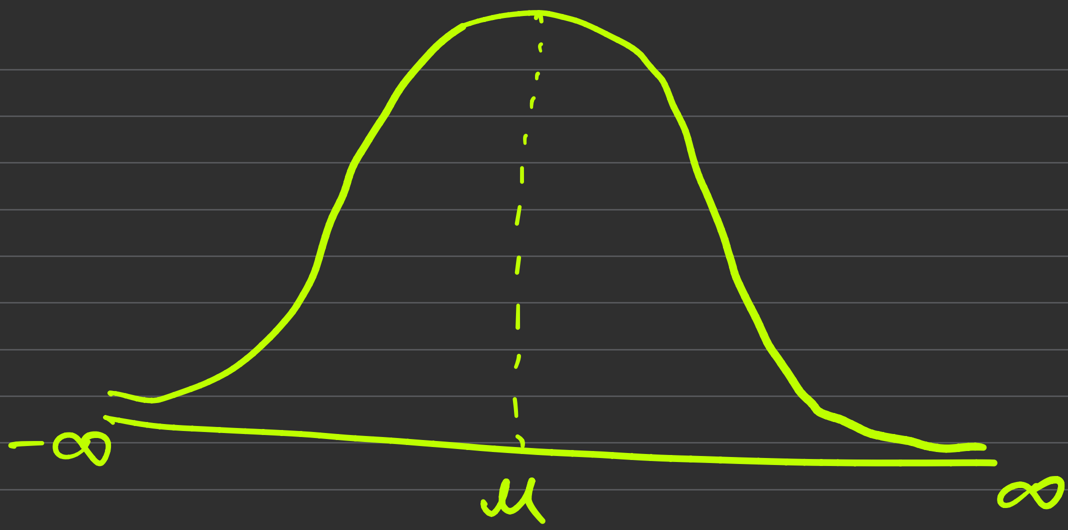


$\mu = 10$
$\sigma^2 = 5$

$X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

for all $-\infty < x < \infty$



→ Normal curve is a symmetrical curve

→ Skewness = Zero

→ Area under this curve is taken 1

$$\int_{-\infty}^{\infty} f(x) = 1$$

$$\int_{-\infty}^{\mu} f(x) = 0.5$$

$$\int_{\mu}^{\infty} f(x) = 0.5$$

→ Both tails of the curve never touches horizontal axis (X-AXIS)

→ In normal Distribution mean = median = mode

→ Mean Deviation = 0.8σ

→ Quartile Deviation (QD) = 0.675σ

$Q_1 = \mu - 0.675\sigma$

$Q_3 = \mu + 0.675\sigma$

median (mean) (mode) $= \dfrac{Q_3 + Q_1}{2}$

→ Point of inflexion

$\mu - \sigma$ & $\mu + \sigma$

→ **Additive Property**

If $x \sim N(\mu_1, \sigma_1^2)$

& $y \sim N(\mu_2, \sigma_2^2)$

Then $x + y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

→ **Standard normal variate**

If $\mu = 0$ & $\sigma = 1$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

This is known as standard normal variate

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

for standard normal variate

$\mu = 0$ & $\sigma = 1$

→ mean = median = mode = 0

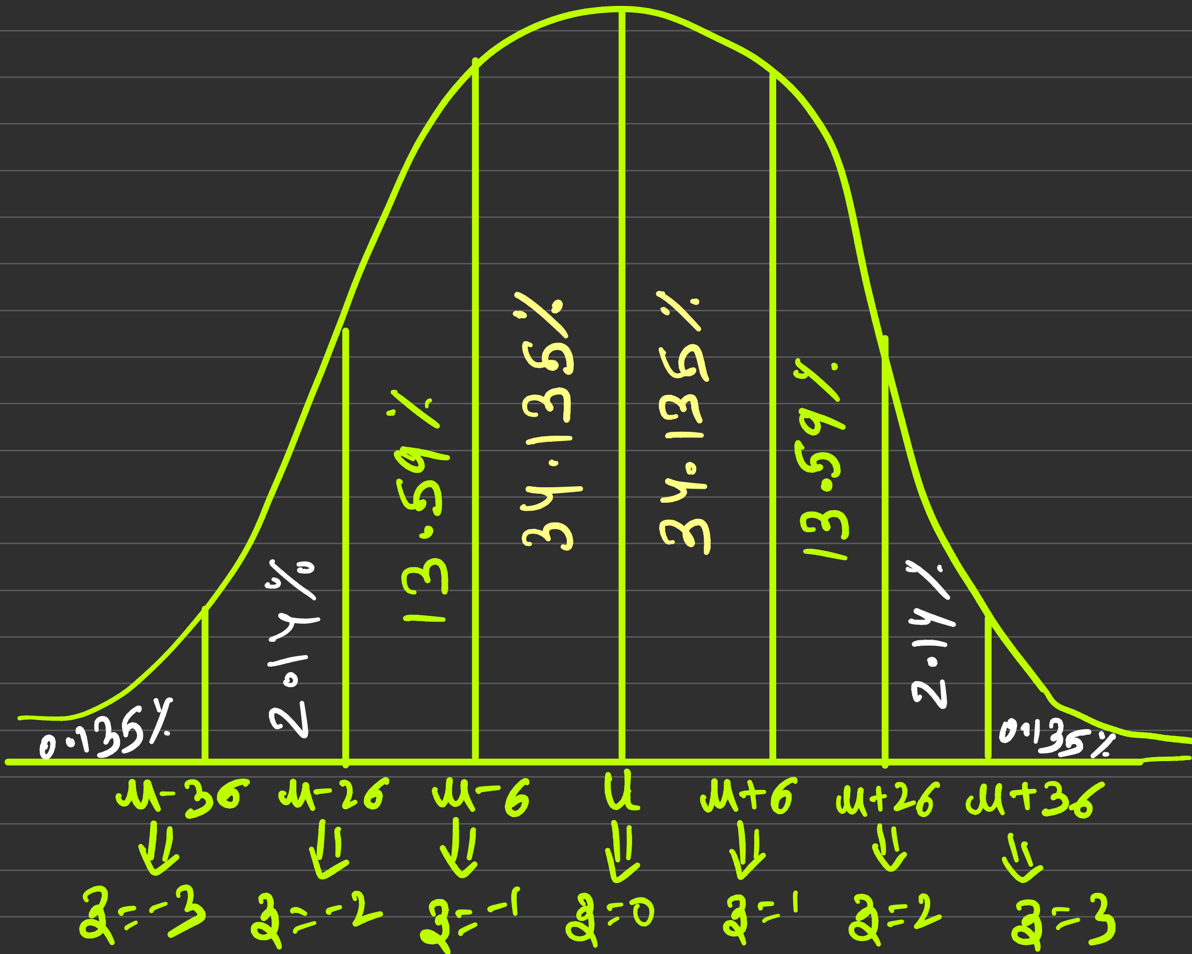
→ MD = 0.8σ = 0.8

→ Q.D = 0.675σ = 0.675

$Q_1 = -0.675$

$Q_3 = 0.675$

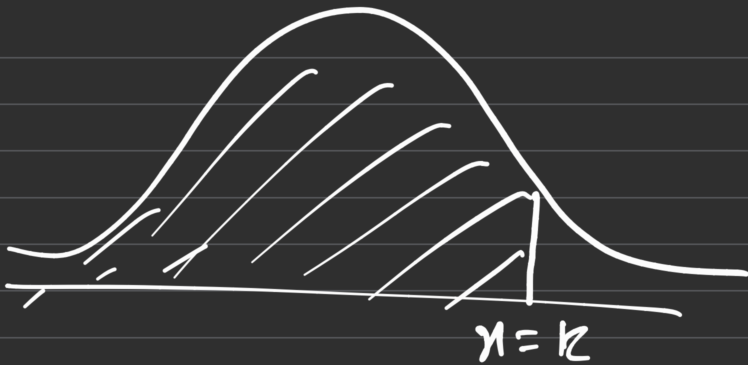
→ Point of inflexion = -1 & 1



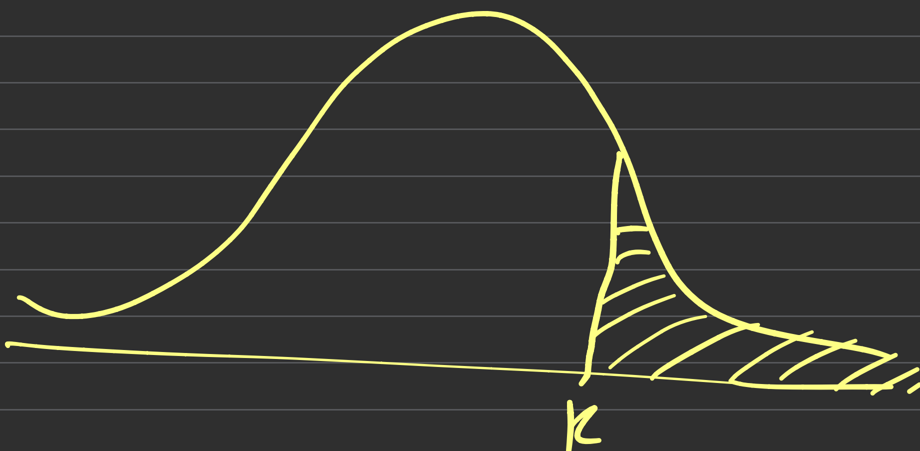
$$P(0<z<1) = \frac{34.135}{100} = 0.3413$$

$$P(-1<z<1) = \frac{68.27}{100} = 0.6827$$

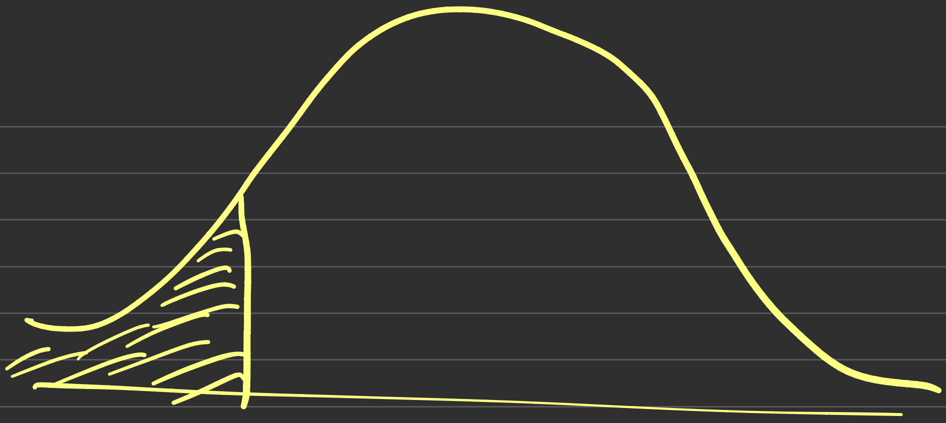
$$P(\mu<z<\infty) = \frac{50}{100} = 0.50$$



$$P(x < k) = \phi(k)$$



$$P(X > k) = 1 - P(X < k)$$
$$= 1 - \phi(k)$$



$$P(x < -k) = 1 - P(x < k)$$

$$\phi(-k) = 1 - \phi(k)$$